

# Role of Trading Volume on the Estimation of Dynamic Extreme Value-at-Risk in Futures Markets<sup>\*</sup>

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## Abstract

The dynamic EVT-based GARCH model has evolved as a preferred approach in the estimation of value-at-risk (VaR), in global financial institutions. Sophisticated risk models also require full information, however, the traditional standard dynamic VaR model failed to account for an important nature of return volatility driven by asymmetric volume changes in the financial markets. The main objective of this study is to investigate whether an incorporation of trading volume improve the accuracy in the estimation of VaR in future markets.

Using alternative dynamic EVT-based GARCH family VaR models including GARCH,

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GJR and EGARCH, over the period from Jan. 1997 to Dec. 2001, the study examine VaRs of three major US futures markets, NASDAQ INDEX, S&P 500 INDEX and NATURAL GAS. Consistent with our a-priori expectation, the finding indicates that the proposed alternative dynamic EVT-based GARCH family VaR models with volumes, in general, outperform traditional dynamic EVT-based VaR models. In particular, GJR+GPD+V is the best model among the others in terms of both rate of violation and RMSE.

**Keywords:** Dynamic EVT-based VaR Model, Trading Volume, Determinant of VaR

**JEL Classification:** G53

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## **I . Introduction**

In recent years, a string of financial failures resulted from inappropriate overwhelming speculation on derivatives and lack of sufficient internal controls have raised considerable concern of market risks among regulators, financial institutions, financial analyst and other participants in the financial markets. For example, in December 1994, Orange County in U.S. had suffered a ever recoded loss of US\$1.6 millions attributed to the unsupervised investment of its treasurer in derivatives securities. In February 1995, a U.K. merchant bank, Barings, was forced into insolvency as a result of huge losses of US\$1.3 billions on its trading in Nikkie stock index future in Japan. In September 1995, a similar incident took place at New York branch of Daiwa Bank resulted from futures trading. In 1997, Eastern Europe and Asia also encountered considerable currency and financial market volatility. This volatility was further magnified throughout 1998 with large losses on Russian bonds as the Russia's ruble depreciated and the price of Russian bonds collapsed. That volatility had forced many large U.S. banks to write off hundred millions of dollars losses on holding Russian government securities.

In response to the above financial disasters, Bank of International Settlement (BIS) revised Basel Accord I in 1998 and required all financial institutions report market risks of

their portfolios and impose capital charges accordingly to restrict the financial institutions from over risk-taking. Subsequently, BIS encourages financial institutions to develop more sophisticated tailor-made model of measuring risks in Basel accord II 2006. The market risk is commonly referred as value change resulting from a change in price, interest rate, market volatility and market liquidity. It can be formally defined as value-at-risk (VaR) which measures the expected maximum loss (or worst loss) over a target horizon within a given confidence interval. The methods to estimate the VaR can be categorized into parametric and non-parametric. Initially, the most popular model was Riskmetric developed by J. P. Morgan Stanley 1994 because of easy use. This traditional variance-covariance-based VaR model, however, fails to account for two important natures of return series: stochastic volatility and fat-tail distribution. More specifically, the traditional method is focused on the confidence interval rather than tail probability of financial return series. Financial literature has well documented that return series in the financial markets are stochastic and fat-tail distributed in nature (Bollerslev, 1986; Bollerslev and Wooldridge, 1992; Bollerslev et al., 1992; Diebold et al., 2000). In fact, risk managers are more concerned with the tail behaviors of market returns. Mathematically, extreme value theory (EVT) approach holds promise for more accurate capturing the extreme quintiles and tail probabilities of financial return series. Nevertheless, traditional EVT techniques assume that financial asset return is independent and identified, and hence fails to account for the other behavior of the asset return series, dynamic clustering of asymmetric stochastic volatility (Diebold et al., 2000; McNeil and Frey, 2000). On the other hand, ARCH/GARCH family models are well recognized as a successful method in capturing the stochastic volatility (Bollerslev, 1986; Bollerslev and Wooldridge, 1992; Bollerslev et al., 1992; Nelson, 1991; Koutmos and Booth, 1995). After the pioneer work of McNeil (1997, 1998) and McNeil and Frey (2000) in the financial risk management, the EVT-based dynamic GARCH model has evolved as a preferred approach in the estimation of VaR (McNeil and Frey, 2000; Longin, 2000; Bystrom, 2004; Gencay and Selcuk, 2004; Fernandez, 2005). Yet, previous works ignore the possible correlation between the financial asset return and trading volume. Numerous financial studies have well documented this important relationship. Clark

(1973) and Epps and Epps (1976) suggested that trading volume is a good proxy for information arrival from the capital market. The hypothesis has been further supported by empirical evidence; Lamoureux and Lastrapes (1990), Kim and Kon (1994), Andersen (1996), Gallo and Pacini (2000) found the same effect for the U.S. stock market; Omran and McKenzie (2000) observed this effect for the U.K. stock market; Bohl and Henke (2003) reported similar evidence for the Polish stock market.

Ying (1966) was the first to provide strong empirical evidence supporting an asymmetric relation between trading volume and price-change. By investigating six series of daily data from NYSE, Ying made the following conclusions: a small trading volume is usually accompanied by a fall in price; a large volume is usually accompanied by a rise in price; and a large increase in volume is usually accompanied by either a large rise in price or a large fall in price. This hypothesis is also documented by Karpoff (1987) in an extensive survey of research into the relationship between stock-price change and trading volume. Karpoff suggests several reasons why the volume-price change relationship is important and provides evidence to support the asymmetric volume-price change hypothesis. His asymmetric hypothesis implies that the correlation between volume and price change is positive when the market trend is going up, but that this correlation is negative when the market trend is downwards. This is again important and highlights that we should not simply add a linear exogenous volume term to the mean equation in a GARCH model for stock returns. To capture the possible nonlinearity we will also consider an asymmetric linear relationship between price (return) and volume, as can be captured by GJR GARCH and EGARCH models.

Departing from traditional work that focused on the contemporaneous relation between return and trading volume, Chordia and Swaminathan (2000) examine the causal relationship and the predictive power of trading volume on the short-term stock return. Their empirical evidence suggests that volume plays a substantial role in the dissemination of national market-wide information. In a dynamic context, Lee and Rui (2002) utilize the GARCH(1,1) model to investigate the relationship between stock returns and trading volume using the New York, Tokyo and London stock markets. Their empirical results suggest that U.S. financial market variables, in particular US trading volume, have extensive predictive power in both the

domestic and cross-country markets, after the 1987 market crash.

Similarly, the asymmetric price change and trading volume relationships also documented by an extensive study in the derivative literature (Cooper, 1999; Fung and Patterson 1999; Lee and Swaminathan, 2000; Bessembinder and Seguin, 1992; Locke and Sayers, 1993; Moosa et al., 2003). In particular, Fung and Patterson (1999) utilize vector autoregressive analysis to examine the relationship of volatility, volume and market depth, and the direction and speed of the information flow between variables in five currency futures markets. The finding suggests that the return volatility is subject to strong reversal effects from trading volume and market depth. In addition, Moosa et al. (2003) employ a bivariate VaR model and find significant mean level asymmetry in the price–volume relationship for the future market in crude oil prices; they did not consider a heteroscedastic model and they enforced the threshold variable to be zero. Aside from the above empirical evidences on the importance of adding trading volume in volatility models based on the market structure of the classical financial literature, the rational of inclusion such trading volume in our conditional volatility models may also stems from the recent behavioral financial literature. Trading volume is an indicator of momentum or sentiment across irrational trader and rational traders and hence is an influential factor to the conditional volatility modeling. (For details, please see McMillan, 2007). The above findings further reinforce our belief that a consideration of trading volume as an explanatory variable might not only add to the understanding of derivatives market behavior in general but improve the accuracy in the estimation of market risk in particular.

The objective of this study is to investigate whether an incorporation of trading volume improve the accuracy in the estimation of VaR in future markets. Consistent with our a-priori expectation, our results indicates EVT-based GARCH family VaR models with volumes, in general, outperform the standard dynamic VaR model and shed the light on the use of trading volume as determinant of dynamic VaR in Futures market.

The remainder of this study proceeds as follows. Section 2 describes the experiment design including the methods and model specifications used in this paper . Section 3 presents an evaluation of alternative models via backtestings. The paper concludes with a summary analysis of the findings in section 4.

## II. Methodology

In the risk management literature, McNeil and Frey (2000) provide an extensive study on the EVT-based models which have been developed to model the tail distribution of financial asset returns. They conclude that traditional EVT-based works by Longin (1997, 2000), McNeil and Saladin (1997) and McNeil (1998) failed to account for the stochastic volatility effect and suggest that a combination of EVT-based model with GARCH family in a dynamic framework will be provide more accurate estimation on the VaR of financial assets. More specifically, McNeil and Frey (2000) filter return series via GARCH model and then utilize a threshold-based EVT technique to estimate VaR in the extreme return series. Recently, Bystrom (2004) expend McNeil's study by adding one extra dimension, the Block Maxima method, to model the tail return distribution and generate similar result. Following Bystrom (2004), we adopt two-stage estimation procedure to estimate the dynamic VaR. In the first stage, we filters different financial time series with a GARCH model. More specifically, the study fits a GARCH-type model to the return data by maximum likelihood. That is, maximize the log-likelihood function of the sample assuming normal innovations; finally, we consider the standardized residuals computed in stage 1 to be realizations of a white noise process, and estimate the tails of innovations using EVT. In particular, we extend the previous work by adding the GJR and EGARCH models to account for asymmetric conditional volatility effect. Furthermore, we formulate the above models adding trading volume as an explanatory variable in the estimation of VaR. The empirical process of this study is presented in the following subsections.

### A. GARCH-type models

In the investment literature, there are several different approaches have been utilized to model financial asset returns. Following the pioneer work of Engle (1982); Bollerslev (1986); Bollerslev and Wooldridge (1992); Bollerslev et al., (1992), the GARCH class model has become a superior model in assessing the stochastic volatility of financial instruments. The

most successful and popular among the others is the GARCH family model with AR-GARCH(1,1) specification (Engle, 1982; Bollerslev, 1986; Gerlach et al., 2006). This popularity is also the motivation behind our choice of GARCH as representing a parametric model for filtering stock returns. As opposed to the EVT-based models described above, GARCH models do not focus directly on the returns in the tails. Instead, GARCH models explicitly model the conditional volatility as a function of past conditional volatilities and returns.

The conditional volatility can be estimated by either a univariate volatility model (single index model as McAleer and da Veiga, 2008a, 2008b), or a multivariate volatility model. The study adopts the univariate formulation in all of our conditional volatility models for two reasons. First of all, the performance of the two models in forecasting the VaR threshold of financial assets is still inclusive. Moreover, McAleer and da Veiga (2008a, 2008b) did a comprehensive empirical study on such issue and suggest that the VaR forecasts are generally found to be insensitive to the inclusion of spillover effects in any of their multivariate models considered. Second, The parsimonious nature of a conditional volatility model is very important to the practitioners (For details please see McAleer and da Veiga, 2008a).

For parsimonious, we adopt standard univariate GARCH(1,1) model to capture the stochastic return volatility of the underlying assets. The AR-GARCH(1,1) model can be defined as follow:

$$\begin{aligned} r_t &= \alpha_0 + \alpha_1 r_{t-1} + \varepsilon_t \\ h_t &= \omega + \alpha \varepsilon_{t-1}^2 + \beta h_{t-1} \end{aligned} \quad (1)$$

where, residual  $\varepsilon_t | \Omega_{t-1} \sim N(0, \sigma_t^2)$  with *mean* = 0, *Variance* =  $\sigma_t^2$ ,  $h_t$  is the conditional variance at time  $t$ , and  $\Omega_t$  is the information set of all information through time  $t$ . Whereas,  $\alpha_0, \alpha_1, \alpha$  and  $\beta$  are parameters to be estimated. When the AR-GARCH model in Eq. (1) has been fitted to data by maximization of the likelihood function, one can estimate (or forecast) dynamic  $\text{VaR}_p$  measures by assuming either the normal distribution or the  $t$  distribution,



multiplying one's estimates (or forecasts) of  $\sigma_t$  with the standard quantiles of each distribution, and finally adding the conditional mean. As cited by previous literature, in comparing to the unconditional EVT-based methods described earlier, the AR-GARCH models have the advantage of producing time varying  $\text{VaR}_p$  measures (Engle, 1982; Bollerslev, 1986; Diebold et al., 2000; Gerlach et al., 2006). Yet, the recent literature further suggest that the return series of current financial markets tends to have not only volatility clustering behaviors but also asymmetric response to the positive and negative shocks (Nelson, 1991; McAleer, 2005; Gerlach et al., 2006; McAleer et al., 2007). Following the novel work of GARCH model by Bollerslev (1986), therefore, a substantial development of extensions on GARCH family models are proposed (McAleer et al., 2007). The most popular extensions are the two asymmetric models, GJR and EGARCH models. McAleer et al. (2007) presents a comprehensive discussion on the similarities and differences among the two extensions and GARCH formulation. According to this literature, the major differences of the three models are on the restriction of response of volatility to positive and negative shocks. Standard GARCH imposes a symmetric response of volatility to the positive and negative shocks, while the latter two formulations accommodate the existence of asymmetric volatility to capture the possible asymmetric response of positive and negative shocks (details as the comprehensive discussion on the theoretical results of GARCH family models by McAleer et al., 2007). In fact, more financial literature has well-documented that the asymmetric type GARCH model provides better estimation of conditional volatility (Nelson, 1991; Gerlach et al., 2006; McAleer et al., 2007; McAleer and da Veiga, 2008a, 2008b). The nature of possible asymmetric response to the positive and negative shocks in the financial asset has been ignored in the previous EVT-based VaR models. This study fills in the gap by adding two asymmetric type GARCH models, GJR and EGARCH.

The model specifications of asymmetric GARCH models are addressed in the following subsections. In GJR framework, the effects of positive shocks (or upward movements in the patent share) on the conditional variance,  $h_t$ , are assumed to be the same as the negative shocks (or downward movements in the patent share) in the symmetric GARCH model. In order to

accommodate asymmetric behaviour, Glosten et al. (1992) proposed the GJR model, for which GJR(1,1) is defined as follows:

$$h_t = \omega + (\alpha + \gamma I(\eta_{t-1}))\varepsilon_{t-1}^2 + \beta h_{t-1} \quad (2)$$

where  $\omega > 0$ ,  $\alpha \geq 0$ ,  $\alpha + \gamma \geq 0$ ,  $\beta \geq 0$  are sufficient conditions for  $h_t > 0$ , and  $I(\eta_t)$  is an indicator variable defined by

$$I(\eta_t) = \begin{cases} 1, & \varepsilon_t < 0 \\ 0, & \varepsilon_t \geq 0 \end{cases}$$

as  $\eta_t$  has the same sign as  $\varepsilon_t$ . The indicator variable differentiates between positive and negative shocks, so that asymmetric effects in the data are captured by the coefficient  $\gamma$ , with  $\gamma \geq 0$ . The asymmetric effect,  $\gamma$ , measures the contribution of shocks to both short run persistence,  $\alpha + \gamma/2$ , and to long run persistence,  $\alpha + \beta + \gamma/2$ .

As for the alternative asymmetric volatility in the conditional variance, the Exponential GARCH (EGARCH(1,1)) model of Nelson (1991), can be formulated as

$$\log h_t = \omega + \alpha |\eta_{t-1}| + \gamma \eta_{t-1} + \beta \log h_{t-1}, \quad |\beta| < 1 \quad (3)$$

According to McAleer et al. (2007), there are five distinct differences between EGARCH and the previous two GARCH models: (1) EGARCH is a model of the logarithm of the conditional variance, which implies that no restrictions on the parameters are required to ensure  $h_t > 0$ ; (2)  $|\beta| < 1$  ensures stationarity and ergodicity for EGARCH(1,1); (3)  $|\beta| < 1$  is likely to be a sufficient condition for consistency of quasi maximum likelihood estimation (QMLE) for EGARCH(1,1); (4)  $|\beta| < 1$  would seem to be a sufficient condition for the existence of moments as the conditional (or standardized) shocks appear; (5) in addition to being a sufficient condition for consistency,  $|\beta| < 1$  is also likely to be sufficient for asymptotic

normality of the QMLE of EGARCH(1,1). Furthermore, EGARCH captures asymmetries differently from GJR. The parameters  $\alpha$  and  $\gamma$  in EGARCH(1,1) represent the magnitude (or size) and sign effects of the conditional (or standardized) shocks, respectively, on the conditional variance, whereas  $\alpha$  and  $\alpha + \gamma$  represent the effects of positive and negative shocks, respectively, on the conditional variance in GJR(1,1). Furthermore, as our a-priori expectation from theoretically ground stated and substantial empirical evidences discussed in the introduction of this study, an incorporation of trading volume into the traditional EVT-based VaR models might improve the accuracy in estimating VaRs. Therefore, we include volume and logarithms of volume as an explanatory variable, denoted as  $V$  and  $\log V$ , into GARCH, GJR and EGARCH model equations to examine such effect, respectively. For example, the conditional variance equation of alternative GARCH model, GJR model and EGARCH model can be formulated in the equation (4) through (6), respectively.

$$h_t = \omega + \alpha \varepsilon_{t-1}^2 + \beta h_{t-1} + \delta V_t \quad (4)$$

$$h_t = \omega + (\alpha + \gamma I(\eta_{t-1})) \varepsilon_{t-1}^2 + \beta h_{t-1} + \delta V_t \quad (5)$$

$$\log h_t = \omega + \alpha |\eta_{t-1}| + \gamma \eta_{t-1} + \beta \log h_{t-1} + \delta \log V_t, \quad |\beta| < 1 \quad (6)$$

Follow Bystrom (2004), we scale our unconditional EVT-based tail estimates with the expected return and volatility. Then, we obtain the forecasts of tail risks that are conditional on the actual market conditions. Thus, after the standardize residual,  $\eta_t$ , from the AR-GARCH model in the first stage in Eq. (1) and the residual distribution quantiles,  $\alpha_p$ , are obtained, we can calculate the forecasted VaR<sub>*p*</sub> quantiles of our return distribution tomorrow as

$$VaR_{t+1,p} = \alpha_0 + \alpha_1 r_t + \sigma_{t+1} \alpha_p \quad (7)$$

where  $\alpha_0 + \alpha_1 r_t$  is the conditional mean and  $\sigma_{t+1}$  is the GARCH forecast of tomorrow's

conditional volatility. Note that the major advantage with first filtering the returns is to obtain IID series which can straightforward to apply the EVT technique. Yet, it is common in the finance literature to apply the EVT technique to financial return series that are known to be non-IID (McNeil and Frey, 2000; Bystrom, 2004).

## B. Modeling the tails of sample return distributions

Traditional methods to estimate the tail distribution under EVT theory can be divided into two groups: the peaks over threshold (POT) method which looks at those events in the data that exceed a high threshold, and block maxima method (BMM) which divides the data into consecutive blocks and focuses on the series of maxima (or minima) in these blocks (Embrechts et al., 1997; Kellezi and Gilli, 2000; McNeil, 1998; Bystrom, 2004). Bystrom (2004) suggests that both BMM and POT generate similar results in estimating and forecasting both conditional and unconditional VaR. Nevertheless, BMM requires long histories for estimation. Therefore, this study adopts POT method to estimate the underlying VaRs.

Under POT method, we collect those returns in the sample series that exceed a certain high threshold,  $u$ , and model these returns separately from the rest of the distribution. Note that the choice of threshold value,  $u$ , is the most important implementation issue in the estimation of EVT. McNeil and Frey (2000) set 10% as the value of threshold,  $u$ , after a careful simulation. Following the study, we set 10% as our threshold value in our empirical implementation at backtesting stage.

As Bystrom (2004), we define a daily return in our data series as  $R$  and assume that it comes from a distribution  $F_R$ . The returns above the threshold  $u$  then follow the excess distribution  $F_u(y)$  that is given by

$$F_u(y) = P(R-u \leq y \mid R > u) = \frac{F_R(u+y) - F_R(u)}{1 - F_R(u)}, \quad 0 \leq y \leq R_F - u \quad (8)$$

where  $y$  is the excess over  $u$ , and  $R_F$  is the right endpoint of  $F_R$ . If the threshold,  $u$ , is high enough, Balkema and de Haan (1974) and Pickands (1975) show that for a large class of distributions,  $F_R$ , the excess distribution,  $F_u(y)$ , can be approximated by the so-called

generalized Pareto distribution (GPD), which can be formulated as

$$\lim_{u \uparrow} \sup_{0 \leq y \leq R_F - u} |F_u(y) - G_{\xi, \alpha}(y)| = 0 \tag{9}$$

$$G_{\xi, \alpha}(y) = \left[ 1 - \left( 1 + \frac{\xi}{\alpha} y \right)^{-1} \right]^{\frac{1}{\xi}}, \text{ if } \xi \neq 0$$

$$G_{\xi, \alpha}(y) = 1 - e^{-\frac{y}{\alpha}}, \text{ if } \xi = 0 \tag{10}$$

for  $0 \leq y \leq R_F - u$ .  $\xi$  is the tail index and for the fat-tailed distributions found in finance, one can expect a positive  $\xi$ .  $\alpha$  is just a positive scaling parameter. Empirically, the tail index,  $\xi$ , as well as the scaling parameter,  $\alpha$ , have to be determined by fitting the GPD to the actual data. These parameters are typically estimated via the maximum likelihood method:

$$\max L_G(\xi, \alpha; y) = \max \sum_i \ln(g_{\xi, \alpha}(y_i)) \tag{11}$$

where  $g_{\xi, \alpha}(y) = \frac{1}{\alpha} \left( 1 + \frac{\xi}{\alpha} y \right)^{-\left(1 + \frac{1}{\xi}\right)}$  is the density function of the GPD distribution if  $\xi \neq 0$  and  $1 + \xi/\alpha y > 0$ . When the GPD distribution and its parameters are estimated, we continue by calculating  $\text{VaR}_p$  quantiles of the underlying return distribution  $F_R$  which can be written as

$$F_R(u + y) = (1 - F_R(u))F_u(y) + F_R(u) \tag{12}$$

Note that  $F_R(u)$  can be written as  $(n - N_u)/n$  where  $n$  is the total number of returns and  $N_u$  is the number of returns above the threshold  $u$ , and that  $F_u(y)$  can be replaced by  $G_{\xi, \alpha}(y)$  (as well as rewriting  $u + y$  as  $x$ ), this expression can be simplified to

$$F_R(x) = 1 - \frac{N_u}{n} \left( 1 + \frac{\xi}{\alpha} (x - u) \right)^{-\frac{1}{\xi}} \tag{13}$$

By inverting this expression, we get an expression for (unconditional)  $VaR_p$  quantiles associated with certain probabilities  $p$ :

$$VaR_p = u + \frac{\alpha}{\xi} \left( \left( \frac{n}{N_u} p \right)^{-\xi} - 1 \right) \quad (14)$$

### III. Empirical Implementations and Backtesting

To valid the appropriateness of alternative risk models in this study, we backtest the underlying models on the historical log return series of three major U.S. futures markets, Nasdaq Index future, S&P 500 Index future and Natural Gas future over the period from Jan. 1997 to Dec. 2001. The plot of sample future indexes and their return series in Figure 1 to 3 shows that an existence of stress, high volatility, in 2001 during the internet bubble. In our implementation, we follow McNeil and Frey (2000) and set 1000 daily returns for the estimation period, an approximation of 4- year duration, and reestimate the model with a one-day sliding window for the testing period from Jan. 2001 to Dec. 2001. More specifically, we reestimate the above various models using the past 1000 days' returns. Using each of the estimates of the underlying models, we produce (1-day)  $VaR_p$  forecasts for the following day.

These  $VaR_p$  forecasts are then compared to the actual return in the particular day. Several procedures can be utilized to valid the accuracy of an EVT-based VaR; however, for practical purpose, we adopt the way enacted by Basel Committee in 1998 (McNeil and Frey, 2000; Longin, 2000). The current verification procedure consists of recording daily violations of the 99 percent VAR over the last year. More specifically, one would expect on average one percent of 250, or 2.5 instances of violations (or exception) over the last year (Jorion, 2002). An exception is said to occur when the actual loss is larger than the forecasted  $VaR_p$ . Therefore, the number of exception is refers to as the number of days when the actual loss is larger than

the forecasted  $VaR_p$ . The Basel committee has regulated that up to four exceptions are acceptable, which defines as green light zone with no corrective action. If the number of exceptions is five or more, the underlying financial institution falls into a yellow or red zone and is subject to progressive penalty.

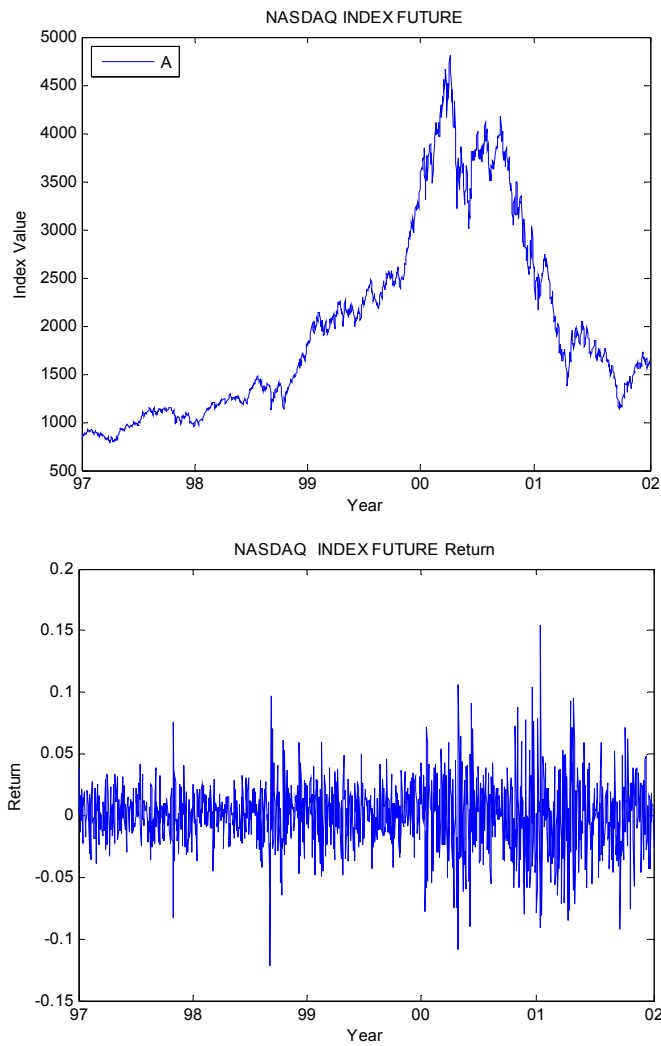


Figure 1 Time Series and Return Series of Nasdaq Index future

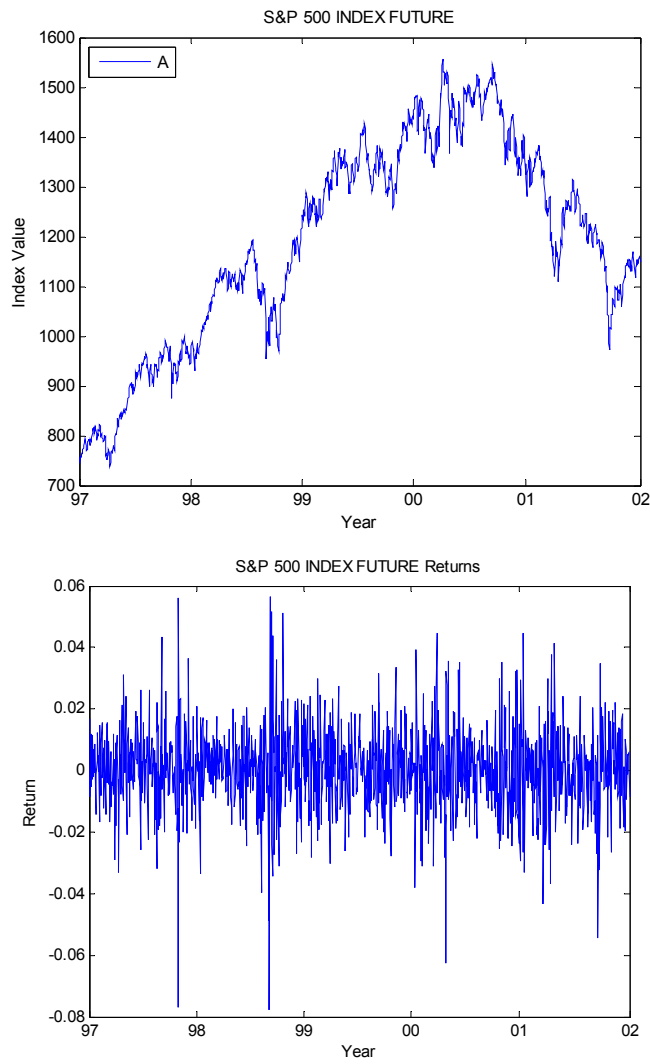


Figure 2 Time Series and Return Series of S&P 500 INDEX future



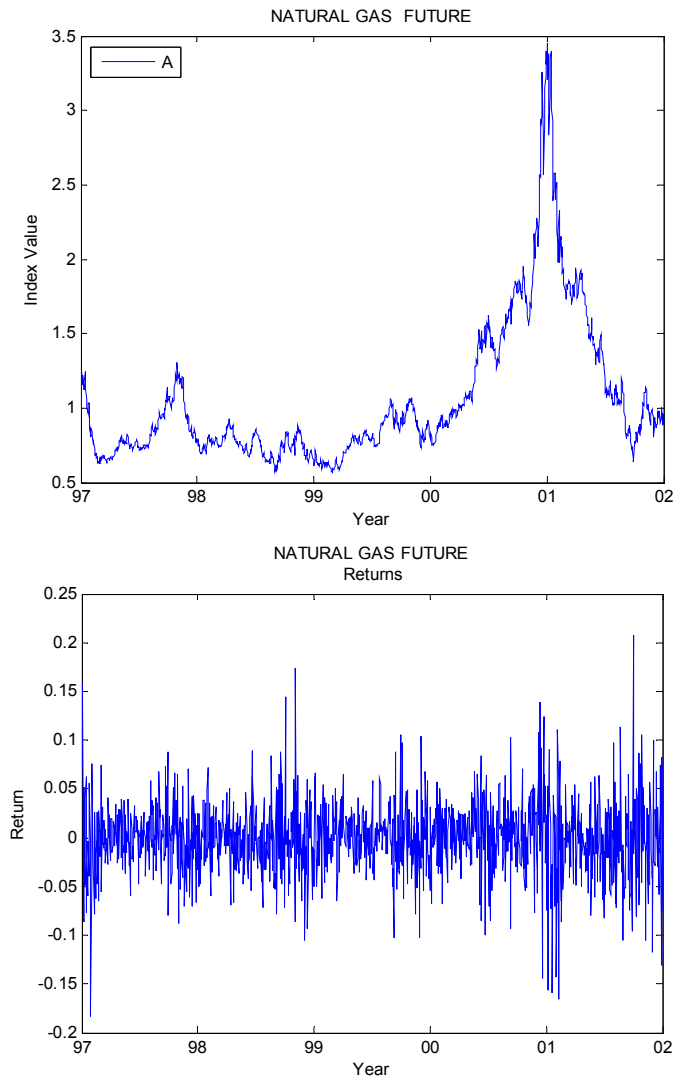


Figure 3 Time Series and Return Series of NATURAL GAS future

To ensure the appropriateness of using GARCH family models, we perform a careful preliminary check on the characteristics of the returns in the sample futures markets. For comparison, key descriptive statistics including mean, medians, maximum, minimum, standard deviations, Skewness, Kurtosis, Jarque-Bera test, results of Augmented Dickey Fuller (ADF) tests, and results of ARCH effect tests, for three sample return series are summarized in table 1. The kurtosis estimates of Nasdaq, S&P 500 and natural gas are 6.98, 5.82 and 6.07, respectively. This highlight that our sample returns are far from normal distributed. The P-values of Jarque-Bera normality tests for the three sample returns further confirm the non-normality at high level of statistical significance. The sample sleekness of Nasdaq and S&P 500 are  $-0.22$  and  $-0.12$ , respectively. This indicates that the asymmetric tails extends more towards negative value than positive value. Overall, high excess kurtosis, high skewness and highly significant Jarque-Bera statistics evidently indicate the sample returns are not normaly distributed. The statistics of the ADF tests on the unit roots indicates that all of sample returns are stationary financial time series at highly statistical significant level. Moreover, the statistics of the ARCH-LM test are 189.69, 71.76 and 35.88, for return series of Nasdaq, S&P 50 and Natural gas, respectively. Their significant p-values show that the three sample returns present the volatility clustering behaviors and hence conclude that the usage of GARCH family model is appropriate. Finally, Table 2 reports the estimation of asymmetric terms in four asymmetric GARCH models. The estimates of all asymmetric volatility models, GJR and EGARCH, in the table are all highly significant at reasonable levels. This conclusion also further confirmed by the sign and bias test. This evidence further lends to the support of using GJA and EGARCH models in the study.

For the tail estimation with POT method, tail index,  $\zeta$ , is estimated by fitting GPD to the sample data. The results are presented in table 3. The estimated tail value ranges from 0.013 to 0.021, 0.014 to 0.027, and 0.074 to 0.101, for Nasdaq, S&P 500 and Natural gas, respectively. These values are greater than zero and highlight that all of them are fat-tailed distributions. This reconfirms the appropriateness of our EVT-based approach.

Table 1 Descriptive statistics and diagnostics of the log daily returns

Future Index	NASDAQ	S&P 500 INDEX	NATURAL GAS
Mean	0.000551	0.000437	0.000630
Median	0.001238	0.000478	0.000000
Maximum	0.167013	0.058176	0.231148
Minimum	-0.115120	-0.074680	-0.167440
Std. Dev.	0.022236	0.013517	0.038116
Skewness	-0.224135	-0.125803	0.237519
Kurtosis	6.986243	5.822081	6.077583
Jarque-Bera Test	1686.211 (0.000)***	420.103 (0.000)***	504.6574 (0.000)***
ADF Unit Root Test	-27.936 (0.000)***	-36.276 (0.000)***	-38.249 (0.000)***
ARCH Effect Test	189.698 (0.000)***	71.764 (0.000)***	35.883 (0.000)***

Notes: 1. *P*-values are in parentheses.

2. \*\*\*, \*\*and\* indicate level of statistics at 1%, 5% and 10% respectively.

Table 2 Estimation of asymmetric terms in alternative models

Future Index	NASDAQ		S&P 500 INDEX		NATURAL GAS	
EGARCH	-0.41400	(0.000)***	0.83059	(0.000)***	0.03773	(0.0293)**
EGARCH + log V	0.06121	(0.000)***	0.94071	(0.000)***	0.98930	(0.000)***
GJR	0.06786	(0.000)***	0.22965	(0.000)***	0.00238	(0.0449)**
GJR + V	0.93908	(0.000)***	0.21660	(0.000)***	0.89408	(0.000)***

Notes: 1. *P*-values are in parentheses.

2. \*\*\*, \*\*and\* indicate level of statistics at 1%, 5% and 10% respectively.

3. The sufficient condition for log-moment has been carefully checked and satisfied.

Table 3 Estimation of tail indices

Future Index	NASDAQ	S&P 500 INDEX	NATURAL GAS
GARCH	0.02116	0.01949	0.10105
GARCH + V	0.02162	0.01707	0.09383
EGARCH	0.01311	0.02220	0.10148
EGARCH + log V	0.01969	0.01469	0.07413
GJR	0.01514	0.02663	0.09984
GJR + V	0.01676	0.02733	0.09760

Note: The sufficient condition for log-moment has been carefully checked and satisfied.

Finally, the relative performance of each model with one 1-day at 99% quantize VaR are summarized as number of exceptions (or violations) in Table 4. All of our proposed alternative models outperform the traditional EVT-based GARCH model, GARCH + GPD, in two stock index future markets. Note that the number of exceptions is 4 for the traditional EVT-based GARCH model in all sample return series. This is fall into the yellow zone and might result into a penalty under the current Basel Accord. Therefore, the performance of traditional EVT-based GARCH model is relative poor in comparison to our alterative models. Especially, those models adding trading volume do improve the accuracy of in all sample markets. In particular, the asymmetric type GARCH models incorporation with trading volume, namely, GJR + GPD + V model provide the best estimate than the others in terms of violation ratios. The same conclusion also applies to the result based on accuracy of forecasting via RMSE in Table5.

Table 4 Number of exceptions of forecasted 1-day 99% VaRs

Future Index	NASDAQ	S&P 500 INDEX	NATURAL GAS
GARCH + GPD	4	2	4
EGARCH + GPD	2	2	4
GJR + GPD	2	2	3
GARCH + GPD + V	2	2	4
EGARCH + GPD + log V	2	2	2
GJR + GPD + V	2	1	2

Table 5 RMSE(%) of forecasted 1-day 99% VaRs

Future Index	NASDAQ	S&P 500 INDEX	NATURAL GAS
GARCH + GPD	4.98	3.99	4.33
EGARCH + GPD	4.41	3.90	4.18
GJR + GPD	4.24	3.96	4.13
GARCH + GPD + V	4.17	3.91	4.21
EGARCH + GPD + log V	3.89	3.65	4.09
GJR + GPD + V	3.78	3.87	3.95

## IV. Summary and Conclusions

Following the pioneer works of Diebold et al. (2000), McNeil and Frey (2000) and Longin (2000), the dynamic EVT-based GARCH family model has evolved as a favored approach in measuring the market risks in the risk management literature. On the other hand, trading volume has well-documented as an important determinant in the assets pricing literature. Nevertheless, previous works in the estimation of market risk ignore the importance and fail to account for the variable in the risk valuation process. The objective of this study is to formulate alternative models which adding trading volume as an explanatory variable in variance equations. In particular, this study extends previous works in two ways: (1) adding asymmetric

GARCH family models to account for the possible asymmetric volatility effects; (2) departing from the traditional EVT-based GARCH family frameworks, this study formulate models that account for the trading volume of futures market. Our empirical implementation proceeded in two-stages. First of all, GARCH family models are established to filter the three sample return series in three U.S. major future markets. Second, we consider the standardized residuals computed in stage 1 to be realizations of a white noise process, and estimate the tails of innovations using POT.

Using alternative dynamic EVT-based GARCH family VaR models including GARCH + GPD + V, GJR + GPD + V and EGARCH + GPD + V, over the period from Jan. 1997 to Dec. 2001, the study examine the value at risk of three major U.S. futures markets, NASDAQ INDEX, S&P 500 INDEX and NATURAL GAS. Consistent with our a-priori expectation, the finding indicates that the proposed alternative dynamic EVT-based Asymmetric GARCH model; in general, outperform the traditional standard dynamic EVT-based GARCH type VaR model. Moreover, an incorporation of trading volume in the model improve the accuracy of VaR estimation. In particular, GJR + GPD + V is the best model among the others in terms of both rate of violation and root mean square errors (RMSE).

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# 交易量在估計期貨市場動態極端 風險值的角色\*

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## 摘要

晚近，動態極值理論基礎 (dynamic EVT-based) 的 GARCH 族群模型已逐漸成為全球主要金融機構在估計其持有資產部位市場風險值 (value-at-risk, VaR) 之較為偏好採用之模型。不過再精緻之模型在實際運用上仍有賴於充份之資訊；依混合分配假說 (mixture distribution hypothesis) 之內涵，交易量 (trading volume) 具有資訊到達 (information arrival) 之功能，因此在理論上，金融資產之交易量可能與報酬波動性具有顯著關聯。然而，現行常用之動態極值風險值模型怠於將金融市場由不對稱交易量變動所引發的報酬波動性納入考量。據此，本研究之目的旨在探討將交易量納入動態極值風險值模型架構，是否能改善期貨市場資產風險值估計的精確性。

實證分析部份，嘗試採用三種常用不同規格之動態極值理論基礎風險值模型加入交易量 (volume, V) 進行剖析。其中，包含 GARCH、GJR、EGARCH 等三組動態極值風

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險值模型。觀察期間自 1997 年 1 月初至 2001 年 12 月底，而以 NATURAL GAS 期貨、NASDAQ INDEX 期貨與 S&P 500 INDEX 期貨等三個期貨資產為樣本，採二階段分析法，首先將交易量納入前述三種常用之動態極值 GARCH 族群模型估計其風險值；其次，運用回溯測試 (back testing) 計算穿越次數，並輔以均方差根 (RMSE) 評估模型的精確性。實證結果，發現有考量交易量之模型精確性均優於傳統動態極值理論法；而在三組模型中，又以 GJR 為最佳。

關鍵詞：動態極值模型、交易量、風險值決定

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